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A Neglected Ricardian Aspect Of Labor Supply

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Abstract
Consider a labor market, or closely related set of such markets, in which a labor service is sold to buyers with differing characteristics, who combine it with other inputs to manufacture consumer's or producer's goods. We assume (and it is a crucial assumption) the service to be tied to, or embodied in, the buyers. We refer.to the service as an embodied labor service, to Indicate the Impossibility of resale. The question arises, how do suppliers distribute themselves among buyers? What factors determine trading with a larger number of demanders more extensively, as opposed to a smaller number more intensively? The problem bears a resemblance to Rlcardo's famous analysis of the cultivation of land, here, an individual buyer represents the Intensive margin, and the number of buyers a supplier deals with, the extensive margin, but the analogous issue remains, whether to trade more intensively or more extensively.

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46

By

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1. Introduction and Background to the Problem.

Consider a labor market, or closely related set of such markets, in which a labor service is sold to buyers with differing characteristics, who combine it with other inputs to manufacture consumer's or producer's goods. We assume (and it is a crucial assumption) the service to be tied to, or embodied in, the buyers. We refer to the service as an embodied labor service, to indicate the impossibility of resale. The question arises, how do suppliers distribute themselves among buyers? What factors determine trading with a larger number of demanders more extensively, as opposed to a smaller number more intensively? The problem bears a resemblance to Ricardo's famous analysis of the cultivation of land, here, an individual buyer represents the intensive margin, and the number of buyers a supplier deals with, the extensive margin, but the analogous issue remains, whether to trade more intensively or more extensively.

The question is not unrelated to topics which have been considered by several works in the literature, but so far as is known, has not been directly attacked. Rosen (1974) examines the matching of buyers with sellers in the context of a general market, using the Hedonic Hypothesis. In his model, producers can continuously vary amounts of objectively measurable goods characteristics; they can perfectly customize their product, and producers and consumers are continuously distributed by these characteristics. Equilibrium in such a market is "spatial" in character. He does not examine what I would term the intensity of this matching process, and its probable relationship to the non-market, or second round, production stressed by Becker (1965).[1] Furthermore, once the decision to trade

[1] We say second round, because the remark applies equally to any production process using a produced means as an input. Production need not be non-market to exhibit this analytical form.

with one buyer more intensively than another is incorporated, a kind of sorting does emerge which does not depend on quality variations. The issue of quality variation is important, because embodied labor markets are susceptible to price discrimination by sellers. Since resale of the service is impossible, only collusion by initial suppliers is required for price discrimination to exist. This possibility gives rise to an identification problem, since to show price discrimination, controls over quality variation must be established.

The organization of this paper is as follows. Section 2 considers the problem in a simple case, where the service is perfectly homogeneous, and no identification problem exists; section 3, in the spirit of Houthakker's (1952) contribution, allows the quality of service to vary according to a single characteristic; section 4 is an extension to the case of n characteristics; section 5 indicates possible applications of the analysis and concludes the paper.

2. The Market with no Quality Variation.

Since in general the production into which the embodied labor service enters as an input may be an item of consumption, or else a producer's good, and since the analysisi of a pure producer's good is simpler, the analysis of this section is broken into two parts. The first part considers the case of a homogeneous service entering into production, the outcome of which has no utility-bearing properties. The second part relates to a mixed consumption-productive case where output of the service is desired because it yields utility directly as well as indirectly through increase of wealth. Also, in view of applications later to be proposed, in each instance let the demanders of the service be ultimate consumers facing symmetric time and goods constraints.

A. A Pure Production Good of the Consumer

Define U as the static utility function of a consumer, Z as the goods consumed and produced by the consumer using own consumption time t_c and market goods X, as in the (by now) familiar model of household production. Hence,

$$U = U(Z) = U[Z(t_c, X)]. \tag{1}$$

Utility is maximized subject to a time constraint, and a goods constraint. Define t as total time, t_w as time spent working, and t_r as residual, or "wasted" time, whose role will shortly be explained. All uses of time must sum to total time, or

$$t = t_c + t_w + t_r. \tag{2}$$

On the other hand, money income is the sum of earnings, the product of the wage W and working time tW, plus non-earned income V. This income is spent on market goods X priced at PX, and other items. For present purposes the other items are the labor service, called L_s, and priced at P_s. Hence, the goods constraint is:

$$t_w W + V = P_x X + P_x L_s. \tag{3}$$

Equations (2) and (3) are not independent and reduce via substitution to

$$tW + V = P_x X + P_s L_s \tag{4}$$

The labor service L_s enters with part of residual time into a production function of the consumer which in general increases effective time and the wage rate. Hence, define:

$$t_r = t_S + t_1, \tag{5}$$

where t_s is the cons-mer's own time which is cooperative with L_s, and t_1 is the pure loss of time. Also define production function for the constraint-expanding commodity S, homogeneous of degree 1,

$$S = S(t_s, L_s). \tag{6}$$

has two productive uses; on the one hand, it can decrease the pure loss of time, expanding the amount of time available for work and consumption; on the other hand, S could raise the wage rate and cause the consumer to be more productive. Therefore, we assume

$$t_1 = t_1(S),\ t_1' < 0 \tag{7}$$

and

$$W = W(S),\ W' > 0 \tag{8}$$

Writing out this simple problem in full, the Lagrangean Function L is

$$L = U[Z(t_c,X)] + (t - t_c - t_w - t_s - t_1[S(t_s,1_s)])\ W\ [S(t_s,L_s)]$$

$$+ V - P_xX - P_sL_s \tag{9}$$

This yields first order conditions (where $U_z = \frac{\partial U}{\partial t}$, etc.)

$$U_zZ_{tc} - W = 0 \tag{10}$$

$$U_zZ_x - P_x = 0 \tag{11}$$

$$- W + t_wW'S_{ts} - t_1'\ WS_{ts} = 0 \tag{12}$$

$$- P_s + t_wW'S_{1s} - t_1'\ WS_{1s} = 0 \tag{13}$$

$$t_wW + V - P_xX - P_sL_s = 0, \tag{14}$$

where primes indicate derivatives.

Equations (12) and (13) show that the marginal product of own time and labor services in producing the producer's good, multiplied by the sum of the marginal contributions to the constraints ($t_wW' - t_1'W > 0$) is equal to the price of each input. Furthermore, the ratio of the marginal products is equal to the ratio of input prices:

$$S_{ts}/S_{1s} = W/P_s. \tag{15}$$

Conditions (12) and (13) imply diminishing marginal effects of own time and the labor service on earnings[2]. They implicitly define optimal values of these variables t_s^* and L_s^* which maximize income[3]. Income is then

$$t_w W^* + V = P_x X + P_s L_s^* \qquad (16)$$

where asterisked variables indicate optimal levels. Utility is then maximized subject to (16), where $t_w = t - t_c - (t_s^* + t_1^*)$, so that t_w and t_c exhaust remaining time, $t - (t_s^* + t_1^*)$.

The usefulness of this analysis hinges on the possibility of examining the effect of shift variables on factor use, particularly of L_s. In addition, examination of the model's structure indicates forces which influence the elasticity of derived demand for L_s. The latter analysis will prove useful in investigating market equilibrium.

We examine first shifts in the wage schedule.[4] The relative demand

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[2]Denoting the value of own time's marginal product as $M_{ts} = (t_w W' - t_1' W) S_{ts}$, $M_{tsts} < W' S_{ts}$, so that marginal costs rise more rapidly than marginal benefits. This is guaranteed for $M_{tsts} < 0$. A similar condition pertains to L_s, so that $M_{ls} = (t_w W' - t_1' W) S_{ls}$ declines, or $M_{lsls} < 0$. These conditions follow from (unconstrained) maximization of the constraint. The remaining second order condition is $(M_{tsts} - W_{tsts}) \cdot M_{lsls} - M_{tsls}^2 > 0$.

[3]Consequently, the pure loss of time is defined, since $t_1 = t_1[S(t_s, L_s)]$, and also W, since $W = W[S(t_s, L_s)]$.

[4]Effects follow from differentiation of (12) and (13), first order conditions for maximization of the constraint, and from differentiation of the production function for S. This yields

$$E_s S_{tsts} \frac{dts}{dW} + E_s S_{lsts} \frac{dls}{dW} + S_{ts} \frac{d\Pi}{dW} = 1$$

$$E_s S_{lsts} \frac{dts}{dW} + E_s S_{lsls} \frac{dls}{dW} + S_{ls} \frac{d\Pi}{dW} = 0$$

$$S_{ts} \frac{dts}{dW} + S_{ls} \frac{dls}{dW} - \frac{dS}{d\Pi} \frac{d\Pi}{dW} = - \frac{dS}{d\Pi} \frac{Es}{W},$$

where Π is the marginal cost of S, $E_S = t_w W' - t_1 W$ is its marginal benefit, and the demand function for S is a function of the difference between marginal costs and marginal benefits: $S = f(\Pi - E_S)$. The third equation represents demand and supply equilibrium. Manipulation of these equations using the first order conditons and the definition of the elasticity of substitution (σ) for CRS production functions, and their solution, yields the results in the text.

The analysis here owes much to Grossman (1972, pp. 90-92). Main differences are the additional effect on wages directly, and the static interpretation which writes the demand for S as a function of the absolute difference in costs and returns.

equation for L_s is

$$\eta_{1sw} = \alpha\sigma + (1 - \alpha)\eta \tag{17}$$

where α is the share of own time in the cost of production of s, σ is the elasticity of substitution, and η is the elasticity of demand for s, positively signed. The output effect is positive, because even though marginal costs shift upward by α, the share of labor in costs, marginal benefits shift upward by a factor of unity.[5] Therefore a rise in the wage rates of working individuals raises the amount of labor service demanded. The effect is greater, the greater are the substitution and output elasticities. The equation for own demand elasticity defined negatively is

$$\eta_{1sps} = - \alpha\sigma - (1 - \alpha)\eta. \tag{18}$$

Therefore, (17) and (18) imply equal but opposite own and cross-price effects on L_s. It can easily be shown that the effect of education, if it acts like a neutral technological improvement, is to increase, leave unchanged, or decrease the demand for the service of the output elasticity exceeded, equalled, or were less than unity (see Michael (1972) for a proof). If education also made own time relatively cheaper, or else increased substitutability between own time and the labor service, it would exert an additional affect, decreasing the demand for the labor service. Other environmental variables other than education (such as an endowed stock of productive capacity) could also prove to be important, and some of these might cause the price of S to rise. In that case the above analysis for education would be reversed; demand for inputs such as L_s would rise if demand for S were inelastic, and so forth.

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[5] This result assumes that the marginal wage, $W' = \frac{dW}{dS}$, shifts upward by the same percentage as the wage itself. thus preserving the elasticity of the wage schedule. Other cases could easily be handled using this framework.

To summarize, the productive model with no quality variation predicts that persons with higher wage rates demand more of the labor service, persons whose demand for output S is inelastic demand less of the labor service the higher their education, and more the lower are endowed stocks of productive capacity. These effects are independent of income per se; converse results occur when demand for S is elastic. Therefore suppliers sort themselves to buyers according to the characteristics mentioned, and not according to property income, for example. The larger demanders tend to be persons with higher wage rates.

The pattern of sorting would be modified by the pattern of price discrimination in monopolized markets for L_s, if larger demanders faced higher prices. In the literature on price discrimination, factors which have been mentioned as playing an important role are income, the wage rate, and the method of fee payment. The first factor serves no role in this analysis; the second would be relevant if higher wage persons had more or less inelastic demands; and the third would be a possibility if different demanders paid their fees in different ways. The effect of higher wages on the own demand elasticity would be to reduce the demand elasticity if substitution possibilities were limited. In the case of fixed proportions and constant demand elasticity,

$$\frac{\partial(\eta_{1sps})}{\partial W} = \frac{-\partial[(1-\alpha)\eta]}{\partial W} = \eta\frac{\partial\alpha}{\partial W} > 0 \qquad (19)$$

since $\frac{\partial\alpha}{\partial W} = \frac{\partial}{\partial W}\left(\frac{Wts}{Wt_s + P_s L}\right) > 0$ if $0 < 1$.[6] Hence the elasticity moves towards zero. In a more general case with constant elasticities,

$$\frac{\partial\eta_{1sps}}{\partial W} = (\eta - \sigma)\frac{\partial\alpha}{\partial W} \qquad (20)$$

[6]Where $E \equiv d\ln$, $\dfrac{E\left(\frac{\partial}{1-\alpha}\right)}{E\left(\frac{W}{P_s}\right)} = \dfrac{E\left(\frac{W_{ts}}{P_s L_s}\right)}{E\left(\frac{W}{P_s}\right)} = 1 + \dfrac{E\left(\frac{t_s}{L_s}\right)}{E\left(\frac{W}{P_s}\right)} = 1 - \sigma \lesseqgtr 0$ as $\sigma \lesseqgtr 1$.

In this case, η_{1sps} becomes more negative if $\eta < \sigma < 1$ or if $1 < \sigma < \eta$. It moves toward zero if $\sigma < \eta < 1$ or if $1 < \eta < \sigma$. Therefore, higher wage persons in this case have more inelastic demands when the output elasticity lies between unity and the elasticity of substitution.[7]

An additional influence on the elasticity (η_{1sps}), it has been claimed [see Fuchs, 1972, pp. 43-44], is the method by which purchases of L_s are financed. For example, use of insurance with imperfect experience rating may result in only a fraction of the price being imposed on the consumer; or else a program of subsidies might create the same effect. However, a difference between the market price of L_s and the price charged to the consumer is not a sufficient condition to affect the demand elasticity. To see this, let P^*_s be the price charged to consumers and P_s be the market price, and let the two be related by a constant β $0 < \beta < 1$. Then $P^*_s = \beta P_s < P_s$, and the relation between the observed elasticity η_{1sps}, and the true elasticity is $\eta_{1sps} = \frac{P^*_s}{1_s \beta} \frac{\partial 1s}{\partial P_s} = \frac{P^*_s}{1_s \beta} \frac{\partial_{1s}}{\partial P^*_s} \frac{\partial P^*_s}{\partial P_s} = \frac{p^*s}{\partial P_s} \frac{\partial_{1s}}{\partial P^*_s} = \eta_{1sps}^*$.

Therefore, whether or not consumers pay the fractional price does not affect the demand elasticity. In order for a <u>reduction</u> in the elasticity to take place, the average effect on price must be less than the marginal effect, so that consumers' price falls in relation to the market price.[8] One payment scheme referred to in the literature assumes a lump-sum payment followed a constant fraction of total fees for an unlimited amount of service. This scheme would fulfill the conditions

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[7] In the case of non-constant elasticities, of course, additional terms would have to be considered. The expression becomes

$$\frac{\partial \eta_{1sps}}{\partial W} = (\eta - \sigma) \frac{\partial \alpha}{\partial W} - [\alpha \frac{\partial \sigma}{\partial W} + (1-\alpha) \frac{\partial \eta}{\partial W}] \qquad (20')$$

[8] Let $P^*_s = f(P_s)$ be the relation between the two prices. Then

$$\eta_{1sps} = \frac{P^*_s}{1_s \frac{f}{P_s}} \frac{\partial_{1s}}{\partial P_s} = \frac{P^*_s}{\frac{1sf}{P_s}} \frac{\partial_{1s}}{\partial P^*_s} \frac{P^*}{P_s} = \frac{P^*_s}{1s} \frac{\partial_{1s}}{\partial P^*_s} \frac{P_{sf'}}{f}$$

so $\eta_{1sps} \lesseqgtr \eta_{1sps}^*$ as $\frac{Pf'}{f} \lesseqgtr 1$. If $\frac{Pf'}{f} < 1$, the marginal consumer's price falls relative to the market price with an increase in the latter.

for a reduction in the elasticity of demand: therefore, even if consumer's price elasticities were identical, so that the term η_{1sPs}^* were identical across persons, η_{1sPs} would be lower for insured than uninsured consumers.

B. A Mixed Producer's and Consumer's Good

Now introduce utility bearing characteristics of the commodity S, and indirectly of the inputs t_s and L_s. The model then becomes (assuming it is time lost which enters the utility function), max

$$U = U\{Z(t_c,X),\ t_1[S(t_s,L_s]\} \tag{21}$$

subject to the constraints (2) and (3). Of the first order conditions, only those pertaining to own time and the labor service are explicitly altered:

$$U_{t_1} t'_1 S_{ts} - \lambda[W - (t_w W' - t'_1 W) S_{ts}] = 0 \tag{22}$$

$$U_{t_1} t'_1 S_{1s} - \lambda[P_s - (t_w W' - t'_1 W) S_{1s}] = 0. \tag{23}$$

The new conditions state that value of marginal product is less than price, due to the utility yield ($U_{t_1} t'_1 > 0$, since time lost is a bad, or $U_{t1} < 0$, and the time loss is reduced, $t'_1 < 0$).[9] Since equation (21) assumes a separable form, the demand functions for inputs depend only on the prices of cooperative inputs and the level of the commodity which they produce [see Strotz (1957, 1959), Gorman (1959)]. The effect of a rise in the consumer's wage is, in elasticity form,

$$\eta_{1sW} = \alpha_s \sigma(\Theta_{1s/W} + \Theta_{ts/W} - 1) + \eta_{ss}\left(\frac{\Pi_s \alpha_s - \Theta_s}{\Pi_S - \Theta_s}\right) + \eta_{sz}\, \alpha_z \tag{24}$$
$$+ \eta_{sF}\left[\frac{W_t}{Wt+V} - \gamma_s\left(\frac{\Pi_s \alpha_s - \Theta_s}{\Pi_s - \Theta_s}\right) - (1 - \gamma_s)\, \alpha_z\right]^{10}$$

Where α_s and α_z the shares of own time in the cost of production of s and z, Θ_{1s}

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[9] Clearly S is no longer produced at a level which will maximize the consumer's wealth.

[10] The derivation of the equation is contained in the Appendix.

Θ_{ts} are value of marginal products of L_s and t_s respectively, η_{ss} and η_{sz} are compensated own and cross elasticities of demand for S, Π_s is the (gross) marginal cost of proudcing S, Θ is the marginal benefit, and $\Pi_s - \Theta$ is the (net) marginal cost.

Finally, η_{sF} is the wealth elasticity of demand for S, and γ_s, $1 - \gamma_s$ are shares of S and Z in expenditures valued at marginal cost. In the mixed model, it is no longer certain that higher wages increase demand for the labor service. The term for substitution in production $[\alpha_s \sigma (\Theta_{1s/W} + \Theta_{ts/W} - 1)]$ is positive by the first order conditions (since $\Theta_{1s/W} + \Theta_{ts/W} - 1 > 0$), but the (net) marginal cost of S may have increased [if $\frac{\Pi_s \alpha_s - \Theta_s}{\Pi_s - \Theta_s} > 0$ or $\alpha_s > \Theta_{s/\Pi_s}$] so that output of S may decrease at constant utility, leading to a decrease in L_s demanded $[\eta_{ss} (\frac{\Pi_s \alpha_s - \Theta_s}{\Pi_s - \Theta_s}) < 0$ if $\frac{\Pi_s \alpha_s - \Theta_s}{\Pi_s \Theta_s} > 0]$.[11] Because S and t are gross substitutes, $\eta_{sZ}\alpha_z > 0$. The final wealth term is again ambiguous, since wealth rises by the share of earnings in all wealth, $\frac{W_t}{W_{t+V}}$, while the rise in (net) costs of the goods, weighted by their share in expenditures is $[\gamma_s(\frac{\Pi_s \alpha_s - \Theta_s}{\Pi_s - \Theta_s}) + (1 - \gamma_s)\alpha_Z]$. Sufficient conditions for L_s to rise with the wage - for η_{1sW} to be positive - are that the net marginal cost fall, so that $\alpha_s < \Theta_s/\Pi_s$, and the net effect on wealth be positive, so that $\frac{W_t}{Wt + V} - \gamma_s(\frac{\Pi_s \alpha_s - \Theta_s}{\Pi_s - \Theta_s}) > (1 - \gamma_s)\, \alpha_Z$. Expressed somewhat differently, the share of own time in the cost of production of S, and the share of Z in the budget must both be "small", in the sense defined above.

The own elasticity of demand is

$$\eta_{1sPs} = - \alpha_s \sigma + (1 - \alpha_s)(\eta_{ss} - \gamma_s \eta_{sF}). \qquad (25)$$

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[11] The condition for S (and L_s) to increase at constant utility is $\alpha_s < \Theta_s/\Pi_s$, or that the ratio of marginal benefits to marginal cost exceed the share of wages in coats. If this is the case, since $E\Theta/EW = 1$ (see the Appendix), the % increase in wages raises marginal benefits to a greater extent than marginal costs.

Again with an eye to studying opportunities for price discrimination, this elasticity would be less in absolute value, the smaller the elasticity of substitution in production, the smaller the compensated substitution term for S, the smaller the share of S in the consumer's (implicit) budget, and the smaller the wealth elasticity. In a constant elasticity case, η_{1sFs} sould be smaller in absolute value the larger is wealth (the wage rate held constant) if $\eta_{sF} < 1$.[12] In this model increases in property can affect the elasticity, and also the level of demand for the service. Hence, so long as S is a normal good, the mixed model predicts a positive effect on the level of L_s of assets of the consumer; the pure production model predicts a zero effect.

Define $\eta_s = -|\eta_{ss} - \gamma_s\eta_{sF}| > 0$, the uncompensated elasticity of demand for S. Then $\eta_{1sPs} = -\alpha_s\sigma - (1 - \alpha_s)\eta_s$, and if σ and η_s were approximately constant,

$$\frac{\partial\eta_{1sPs}}{\partial W} = (\eta_s - \sigma)\frac{\partial\alpha_s}{\partial W}. \tag{26}$$

For this special case, the same analysis pertains (as on pp. 7-8) of the effect of a wage increase on the elasticity. η_{1sPs} becomes more inelastic with a higher wage if $\eta_s < \sigma < 1$, or $1 < \sigma < \eta_s$, and more elastic if $1 < \eta_s < \sigma$ or $0 < \eta_s < 1$. We cannot say in this case whether higher wage persons have more inelastic demands, even if we know σ lies between 1 and η_s, because a rise in the wage need not indicate a larger purchase [see the discussion of equation (24)].

Once again, if a payment system were introduced which charged a declining fraction of the price of a unit of L_s as L_s increased, demand for L_s would be more inelastic for those included in the system than those not included.

The principal differences between the mixed and pure models, to summarize this

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[12] $F \cdot \dfrac{\partial\eta_{1sPs}}{\partial F} = \eta_{sF}(\eta_{sF} - 1)\gamma_s \gtrless 0$ as $\eta_{sF} \gtrless 1$. This assumes all elasticities, σ, η_{ss}, and η_{sF}, are constant.

discussion, are that in the mixed model, a rise in the wage need not increase level of demand for the service, a rise in level of wealth, unaccompanied by a rise in the wage, typically increases level of demand for the service, and possible also its elasticity. In the pure producer's good model, a rise in the wage must increase level of demand, and pure income effects play no role. Conditions for price discrimination are correspondingly affected.

2. The Model With Quality Variation in A Single Dimension

Now introduce variation in quality of the labor service; better or worse service is available at the appropriate price. This section adopts a modification of the consumer's budget line introduced by Rosen (1974), which is itself an adaptation of Houthakker's (1952) work on quality variation. Let q be an objective measure of the quality of L_s, and let the price of a unit of L_s depend on q in the relation $P_s = P_s(q)$.[13] The production of the commodity S is assumed to depend on q as well as on own time t_s and the labor service L_s, so $S = S(L_s,q,t_s)$, where S is homogeneous of degree 1. Therefore we rewrite the budget constraint with a variable price p_s, and an additional argument in the production function for S. For simplicity neglect the role of S as a consumer good; then the Lagrangean function is

$$L = U[Z(t_c,S)] + \lambda\{(t - t_c - t_s - t_1 [S(t_s, q, L_s)]) W[S(t_s, q, L_s)] + V - P_x X - P_s(]) L_s\} \quad (27)$$

Simplify momentarily by letting $P_s(q) = a \cdot q$. First order conditions for wealth maximization are

$$(t_w W' - t_1' W) S_{t_s} - W = 0 \quad (28)$$

[13] A linear formulation would write $p(q) - a \cdot q$. There is no reason to expect linearity, unless arbitrage across qualities is possible; but if these services are tied bundles, only separable at a cost, resale activity does not occur.

$$(t_w W' - t_1' W) S_{l_s} - aq = 0 \tag{29}$$

$$(t_w W' - t_1' W) S_q - a_{l_s} = 0 \tag{30}$$

Differentiation of these conditions and the production function - demand function equilibrium condition yields the demand functions.[14] The effect of a change in the wage is

$$\eta_{lsw} = \alpha_{ts} \tilde{\sigma}_{tsls} + (1 - \alpha_{ts}) \eta \tag{31}$$

where α_{ts} is the share of ts in the cost of productuion of S, η is the elasticity of demand for S (defined positively), and $\tilde{\sigma}_{tsls}$ is the (adjusted) partial elasticity of substitution between own time and labor service of constant quality. The elasticity is adjusted by the presence of the price terms $\frac{a}{\Pi}$; if it is positive, and own time and the labor service are substitutes, persons with higher wages demand more units of the labor service for any quality, and the results of the previous section continue to apply. However, were the two inputs complementary, the conclusion would no longer continue to hold.

Analysis of shift factors is also parallel to the previous section. The wage elasticity of the quality of medical care is

$$\eta_{qw} = \alpha_{ts} \tilde{\sigma}_{tsq} + (1 - \alpha_{ts}) \eta, \tag{32}$$

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[14]The equation system obtained is

$$\eta S \frac{\partial \Pi}{\partial W} \frac{1}{\Pi} + S_{ts} \frac{\partial ts}{\partial W} + S_{ls} \frac{\partial ls}{\partial W} + S_q \frac{\partial q}{\partial W} = \eta \frac{S}{W}$$

$$S_{ts} \frac{\partial \Pi}{\partial W} \frac{1}{\Pi} + S_{tsts} \frac{\partial ts}{\partial W} + S_{tsls} \frac{\partial ls}{\partial W} + S_{tsq} \frac{\partial q}{\partial W} = \frac{1}{\Pi}$$

$$S_{ls} \frac{\partial \Pi}{\partial W} \frac{1}{\Pi} + S_{tsls} \frac{\partial ts}{\partial W} + S_{lsls} \frac{\partial ls}{\partial W} + (S_{lsq} - \frac{a}{\Pi}) \frac{\partial q}{\partial W} = 0$$

$$S_q \frac{\partial \Pi}{\partial W} \frac{1}{\Pi} + S_{tsq} \frac{\partial ts}{\partial W} + (S_{lsq} \frac{a}{\Pi}) \frac{\partial ls}{\partial W} + S_{qq} \frac{\partial q}{\partial W} = 0$$

The details of the manipulations are somewhat lengthy and are outlined in the second section of the Appendix. This system is a generalization of the model in section 2, fn. 4.

and analagous considerations apply: higher wage persons would clearly demand higher quality services if the adjusted partial elasticity of substitution between own time and quality of service were positive, but might not if the two were complements. The wage elasticity of the quality of the service would exceed or be less than the quantity wage elasticity as $\tilde{\sigma}_{ts}q \gtrless \tilde{\sigma}_{tsls}$.

Consider now the effect of a change in the common price parameter a, pertaining both to L_s and q. Examination of the first order conditions shows that a change in a affects the price of both L_s and q, and this is a feature of quantity-quality models generally. The quantity elasticity of the service is

$$\eta_{ls}a = \alpha_{ls}(\tilde{\sigma}_{lsls} - \eta) + \alpha_q(\tilde{\sigma}_{ls}q - \eta). \qquad (33)$$

Because $\alpha_{ls} + \alpha_q = 1 - \alpha_{ts}$, and $\alpha_{ls}\tilde{\sigma}_{lsls} = -(\alpha_q\ \tilde{\sigma}_{ls}q + \alpha_{ts}\ \tilde{\sigma}_{lsts})$ [see Allen (1938), p. 502 for a proof] (33) can be further simplified to

$$\eta_{lsa} = -\alpha_{ts}\ \tilde{\sigma}_{lsts} - (1 - \alpha_{ts})\ \eta. \qquad (33)'$$

Thus the effect of a rise in the common price parameter a is unambiguously negative if L_s and t_s are substitutes; a perverse case ($\eta_{lsa} > 0$) might arise if $\tilde{\sigma}_{lsts} < 0$ and $\tilde{\sigma}_{lsts} < -\frac{(1-\alpha_{ts})}{\alpha_{ts}}\eta$. The analogous elasticity for q is

$$\eta_{qa} = \alpha_q\ (\ \tilde{\sigma}_{qq} - \eta) + \alpha_{ls}(\tilde{\sigma}_{ls}q - \eta) \qquad (34)$$

$$= -\alpha_{ts}\ \tilde{\sigma}_{ts}q - (1 - \alpha_{ts})\ \eta, \qquad (34)'$$

thus raising the same issue about sign of the partial elasticity. If we make the plausible assumptions that $\tilde{\sigma}_{lsts} > 0$ and $\tilde{\sigma}_{qts} > 0$, so that both the quantity and quality of the service are substitutes for own time, both η_{lsa} and η_{qa} are negative.

What factors might affect the own demand elasticity? Assuming constant substitution and demand elasticities, the effect of a change in the wage is

$$\frac{\partial|\eta_{1sa}|}{\partial W} = (\tilde{\sigma}_{1sts} - \eta)\frac{\partial\alpha_{ts}}{\partial W} = (\tilde{\sigma}_{1sts} - \eta)(1-\alpha_{ts})(1-\bar{\sigma}_{ts})\frac{Wts}{S}\frac{1}{W} \tag{35}$$

where $\bar{\sigma}_{ts} = \frac{\alpha ls}{1-\alpha_{ts}}\tilde{\sigma}_{tsls} + \frac{\alpha q}{1-_{ts}}\tilde{\sigma}_{ts}q.$[15] Therefore, $\frac{\partial|\eta_{1sa}|}{\partial W} \lessgtr 0$ and the demand elasticity decreases with the wage if $\tilde{\sigma}_{tsls} > \eta$ and $\bar{\sigma}_{ts} < 1$ or $\tilde{\sigma}_{tsls} < \eta$ and $\tilde{\sigma}_{ts} > 1$. The analogous condition for η_{qa} is

$$\frac{\partial|\eta_{qa}|}{\partial W} = (\tilde{\sigma}_{tsq} - \eta)\ (1 - \alpha_{ts})(1 - \bar{\sigma}_{ts}) \cdot \frac{Wts}{S\Pi}\ \frac{1}{W} \tag{36}$$

allowing similar conclusions to be drawn. The discussion concerning other factors which affect the price elasticity is largely parallel to similar passages in the preceding section: the payment of fees through prepayed insurance will lower price elasticity if the fraction of price imposed on the consumer declines as the price increases, and so on. Changes in income with no rise in the wage have no effect either on the level or elasticity of demand.

The main differences between the present model and the earlier one consist in the replacement of the elasticity of substitution with partial elasticities, and the possibility of complementarity among the purchased inputs in quality and quantity. Furthermore, the interactive relation in the budget between quality and quantity implies cross-effects on the quantity and quality of the service which can result in a rise in the demand for each under unusual circumstances, since the price of the other input also is rising. The model can be generalized slightly to allow for non-linearity of the joint price. Instead of $P_s(q) = aq$, simply write $P_s(q)$, where $P_s'(q) > 0$, and $P_s''(q) \gtrless 0$. This affects the second-order

[15] Differentiate α_{ts}, remembering that $\alpha_{ts} = \frac{W}{\Pi}\frac{ts}{S}$, and ts/S does not depend on the level of output S for a CRS production function.

conditions in a trivial fashion.

Finally, since firms also will differ in the quality of labor services they supply, quality sorting - matching of buyers and sellers in the quality dimension - also will emerge. The intensive margins in this case, which apply to an individual buyer, are two. The quality margin determines the choice of firm, or supplier of the service, since firms operate along a quality spectrum. The quantity margin determines what fraction of its resources a firm allots to a demander.

The fact that quality can vary implies that identification of price of the service is a problem. Given a quality index, separation of price from quality might proceed as follows. Regress the observed price of the service on the quality index; and then calculate residuals of the difference in observed and predicted prices. The difference represents an estimate of price variation among demanders, and can be related to factors which have been mentioned as possible causes of discrimination in a secondary regression. This then would estimate the importance and direction of these factors in price discrimination.

4. Generalization to N Quality Dimensions.

Continue to utilize the pure production model, but now allow for N distinct characteristics (quality dimensions) of the service. Write $P_s(q_1, q_2 \ldots, q_N, \beta)$ where β is a shift variable, $\frac{\partial P_s}{\partial q_i} > 0$, and $P^s_{ij} = \frac{\partial P_s}{\partial_{qi} \partial_{qi}} \neq 0$, necessarily.

First-order conditions of the model in section 3 with P_s rewritten according to the above, are

$$(t_w W' - t_1' W) S_{ts} - W = 0 \tag{37}$$

$$(t_w W' - t_1' W) S_{1s} - P_s = 0$$

$$(t_w W' - t_1' W) S_{qi} - \frac{\partial P_s}{\partial_{qi}} L_s = 0.$$

$$i = 1, 2, \ldots., N.$$

Therefore, all previous manipulations would continue to hold, except that now there N + 2 cross-partial elasticities of substitution, and in an analysis of the effect of exogenous shifts in β, there are N − 1 interactive terms to consider, instead of 1, as in the preceding section. Otherwise nothing significant in the model is changed.[16]

5. Applications of the Theory.

In this section we consider two potential applications of the approach. The markets for services of lawyers and physicians exemplify the necessary conditions for its relevance, since the service is combined with characteristics of clients, to produce an outcome which is in each case beneficial to them. Each market is characterized by tying of the service to buyers, the consequence being impossibility of resale, which given a degree of monopolization implies the likelihood of price discrimination. These markets also exhibit considerable variation in quality of service, so that the price identification problem, and the quantity-quality tradeoff are important to explicitly consider.

There is some reason to suppose that the specification of marginal benefits is correct for these applications. Healthier persons are more productive and earn higher wages than less healthy ones; therefore it is only necessary to establish a positive link between health and medical care to show $S_{1s}W' > 0$, since $W' > 0$. Clearly the same reasoning applies to days lost: healthier persons lose less time, so $t_1' < 0$, and it only remains to show $S_{1s} > 0$.[17]

Lawyers too, by decreasing prison sentences of their clients reduce days lost, or increase "free" days (see Landes, 1973 for a discussion). Furthermore,

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[16] It must be remembered that the second-order conditions contain terms in P_{ij} in this general case.

[17] Fuchs (1972) stresses a small but significantly positive effect of L_s.

embodied labor services of buyers has been explored. Such markets are prone to price discrimination. Conditions for level and elasticity of quantity and quality of demand received the most emphasis. Empirically the main contribution is to suggest directions of price discrimination by buyers and a method for separation of quality from price variation of the labor service.

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